



Determining the Basic Variables for Diagnosis of Patients With Thyroid Gland By Using S/N Ratio Methods

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Article info

Original: 21 Jan. 2015
Revised: 27 Mar. 2015
Accepted: 19 Apr. 2015
Published online:
20 Sep. 2015

Key Words:

Hotelling².chart
Signal-to-noise ratio
S/N.

Abstract

There are many situations in which the simultaneous monitoring or control of two or more related quality characteristics is necessary. Multivariate analysis utilize the additional information due to the relationships among the variables and these concepts may be used to develop more efficient control charts than the simultaneous operation of several univariate control charts. The most popular multivariate statistical process control SPC are the Hotelling's T^2 chart. In this paper the S/N ratio is analyzed and response effect level are calculated with optimizing the response effect level And to explain which of variable is important and necessary to test the chemical analysis in Thyroid gland.

Introduction

Quality control via the use of statistical methods is a very large area of study in its own right Statistical process control (SPC) is important statistical technique to analysis the data in the quality control field, In 1926 Dr. Shewhart published the paper on "Quality Control Charts". It is a fact of life that most data are naturally multivariate .The first original study in multivariate quality control was introduced by Hotelling (1947). The Hotelling T^2 distance is a measure that accounts for the covariance structure of a multivariate normal distribution.

This paper covers both the motivation of multivariate quality control and a discussion of some of the techniques currently available and to analysis the normal and abnormal variable.

The Signal –to –Noise Ratio S/N is analyzed and response effect level are calculated with optimizing the response effect level .The abnormal data are analyzed to identify the useful variables of Thyroid gland in male and female, and to show which variable has a good effective on the thyroid gland, and determine which of variable is important must test for patient.

1-The Hotelling T^2 Control Chart:

The Hotelling's T^2 chart is the most familiar multivariate process monitoring and control procedure for monitoring the mean vector of the process[4]. Multivariate control chart has based heavily upon Hotelling T^2 distribution [7] Then the real population value is unidentified, the Hotelling's T^2 control charts will be used . The first original study in multivariate quality control was introduced by Hotelling (1947). He

applied his procedures to bombsight data during World War II. Before this study, he wrote a paper on T²-test procedures for multivariate population in the 1931.[8]

This quantity measures the squared standardized distance from X to the mean (μ), where by the term “standardized” it means that the distance is expressed in standard deviation units. The same approach can be used in the multivariate normal distribution case. Suppose that we have (p) random variables, given by :

$\bar{X}_1, \bar{X}_2, \dots, \bar{X}_p$. Arranging these variables in a p-component vector.

Let the variances and covariance of the random variables in X be contained in a p × p covariance matrix Σ. The main diagonal elements of Σ are the variances of the X’s and the off-diagonal elements are the covariance. These random variables can be organized as follows:

$$\bar{X}_k = \begin{bmatrix} \bar{X}_{1k} \\ \bar{X}_{2k} \\ \vdots \\ \bar{X}_{pk} \end{bmatrix} \quad u = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix} \quad k=1,2,3,\dots$$

Vector μ is population mean vector .

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1p} \\ & \sigma_2^2 & \cdots & \sigma_{2p} \\ & & \ddots & \vdots \\ & & & \sigma_p^2 \end{bmatrix} \quad \dots 1$$

Since Σ is the matrix of variance and covariance. The matrix can be combined to define the following random variable:

$$\chi_k^2 = n (\bar{X}_k - \mu)^t \Sigma^{-1} (\bar{X}_k - \mu) \quad \dots 2$$

Where $(\bar{X}_k - \mu)^t$ is the transpose of $(\bar{X}_k - \mu)$ and Σ⁻¹ is the inverse of Σ . This random variable is one-dimensional.

Suppose that two quality characteristics X1 and X2 are jointly distributed according to be variety normal distribution, this formulas use to compute the Hotelling’s T² chart.

$$\chi_k^2 = \frac{n}{\sigma_1^2 \sigma_2^2 - \sigma_{12}^2} \left[\sigma_2^2 (\bar{X}_{1k} - \mu_1)^2 + \sigma_1^2 (\bar{X}_{2k} - \mu_2)^2 - 2\sigma_{12} (\bar{X}_{1k} - \mu_1)(\bar{X}_{2k} - \mu_2) \right] \quad \dots 3$$

Where σ_{12}^2 is the square of the covariance σ_{12} and the upper control limit applied to the preliminary samples is given by: [1]

$$UCL = \frac{p(m-1)(n-1)}{m(n-m-p+1)} F_{\alpha, p, m(n-m-p+1)} \quad \dots 4$$

2- Signal –to –Noise Ratio S/N

The signal to noise tries to capture the magnitude of true information after making some adjustment for uncontrollable variation. In the case of multivariate diagnosis, S/N ratio is defined as the measure of accuracy of the measurement scale for predicting the abnormal conditions S/N ratio is expressed in decibel(dB). It is important to identify a useful set of variables that is sufficient to detect the abnormal, It is also important to assess the performance of the system under different condition .[9]

The signal–to–noise ratio (S/N) is a measurement scale that has been used in the several fields, as used in the communication industry and applied it for the evaluation of measurement systems as well as for the function of products and processes[10].

The general idea is to use an (S/N) that is appropriate for a particular situation. The S/N, which is to be maximized, is presumed to be a logical estimator of some performance measure, and to minimize the number of variables. A higher value of the S/N ratio means lower error of prediction .[6]

To estimate S/N ratios of different runs using the Mahalanobis distance (MD) was introduced by P. C. Mahalanobis in 1936 . It is based on correlations between variables by which patterns can be identified and analyzed.[3] Mahalanobis Taguchi system calculated for the jth observation in a sample size n with k variables using the following formula.

$$MD_j = \frac{1}{K} Z_j^T A^{-1} Z_j \quad \dots\dots 5$$

Where $(Z_1, Z_2, Z_3, \dots, Z_k)$ are the set of standardized vectors equation:

$$Z_{ij} = \frac{(X_{ij} - m_i)}{S_i} \quad \dots\dots 6$$

where,

- m_i : mean of the attribute,
- S_i : standard deviation of attribute,
- Z_i : standardized variables, and
- X_i : normal observations.

The standardized vector is obtained from the standardized values of $X_i, i=1, 2, \dots, k$, Where A^{-1} is the inverse of the correlation matrix, which contains correlation coefficients between the variables, T is the transpose of the standard vector and k is the number of data sets. [2][12]

The S/N ratio (measure of accuracy of the measurement scale) is used to identify the useful variables .S/N ratio are used as a basis of a “Larger-the better” criterion. Larger-the-better-type S/N ratios corresponding to run of an orthogonal array is. [9]

$$S / N = -10 \text{ Log} \left(\frac{1}{K} \sum_{j=1}^k \left[\frac{1}{(Y_j^2)} \right] \right) \quad \dots\dots 7$$

The useful set of variables is obtained by the evaluating the “gain” in S/N ratio .The signal-to-noise (SN) ratio is used to assess the effectiveness of a system. Moreover, which observations outside of the control chart used, usually abnormal or test observations is used to optimize a system of multivariable diagnosis.

The gain in S/N ratio is the difference between the average S/N ratio .If the gain is positive when the variables is useful, and to define the gain by the following equation. [11] [5]

An average S/N ratio at level-1 and level-2 is obtained for each feature. Subsequently, The gain in average S/N ratio values is obtained as, [2][6]

$$\text{Gain} = (\text{Average S/N Ratio})_{\text{Level 1}} - (\text{Average S/N Ratio})_{\text{Level 2}} \quad \dots\dots 8$$

3- Practical analysis:

This paper concludes the result of medical data from chemical analysis laboratory (Naw in Sulaimani) on Thyroid gland presented by laboratory from chemical analysis of sample size 240 test.

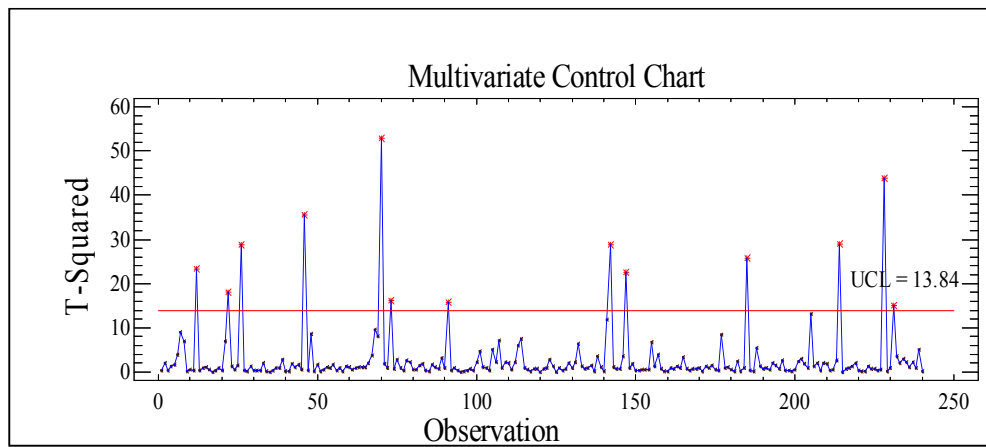
The patients selected consist of (112) male and (128) female each test contain three variables.

1-Triiodothyronine (T3), 2-Thyroxine (T4), and 3-Thyroid Stimulating Hormone, T.S.H .

In this case, gender was a qualitative variable a number for each class is assigned, say number (1) for female and (10) for male as show in table (1 and 7).

Table(1) Define the variable

#	Notation for analysis	Variables
1	X1	Gender
2	X2	Triiodothyronine Test (T3)
3	X3	Thyroxine –Test (T4)
4	X4	Thyroid stimulating hormone –Test (T.S.H)



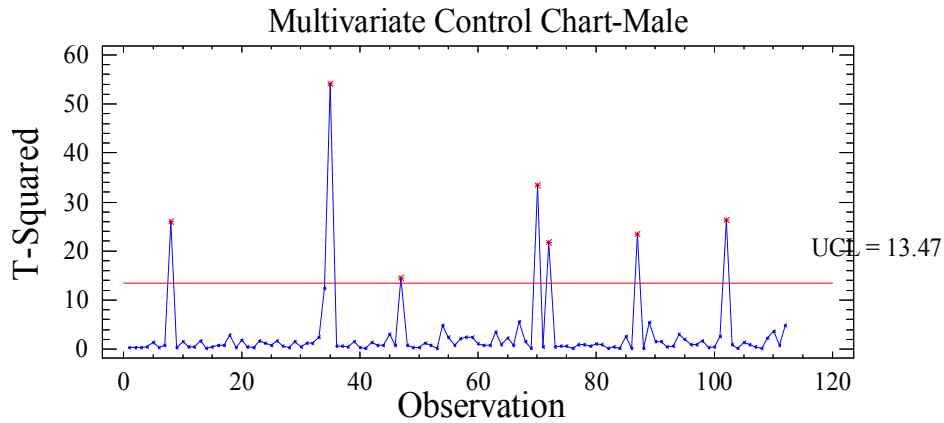
Fig(1) Hotelling's T^2 -Chart of 240 Observation

Table (8) and Fig(1) shows Hotelling's T^2 chart of multivariate process of three variables (T3,T4,T.S.H) , the range of the value of variable T^2 are between (0.0352 and 52.847).

The Upper Control Limit is (13.48) there are (13 points) out of upper control limit these points are (12, 22, 26, 46, 70, 73, 91, 142, 147, 185, 214, 228, 231) from these points (6) female and (7) male ,with mean and variance (2.98 , 6.67), and the mean and variance without abnormal points is (1.6, 2.1) the range of abnormal between (15.01 – 52.85) the abnormal data as shows in table(2).

Table(2) Abnormal Observation Female and Male

# observation	T^2 Female	# observation	T^2 male
12	23.28	26	28.69
22	18.08	70	52.85
46	35.51	91	15.77
73	16.08	142	28.87
228	43.85	147	22.65
231	15.01	185	25.72
		214	28.96



Fig(2) Hotelling's T^2 Chart of Male Observation

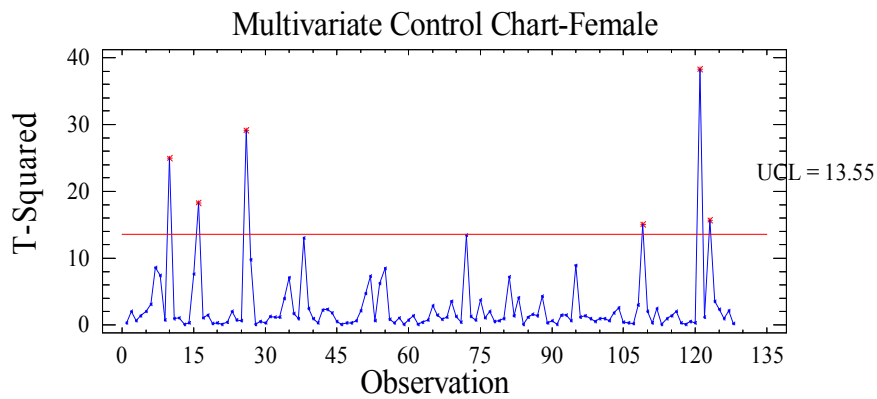


Fig (3) Hotelling's T^2 Chart of Female Observation

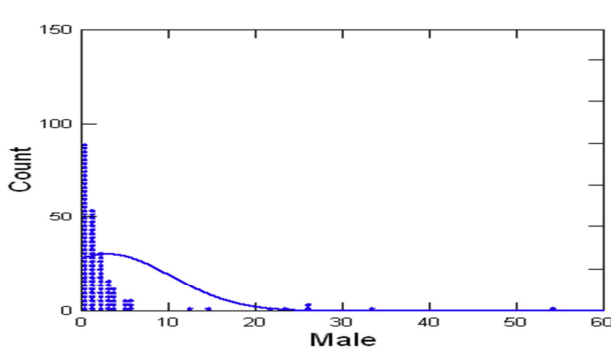


Fig.(4) Histogram Chart for T^2 -Male

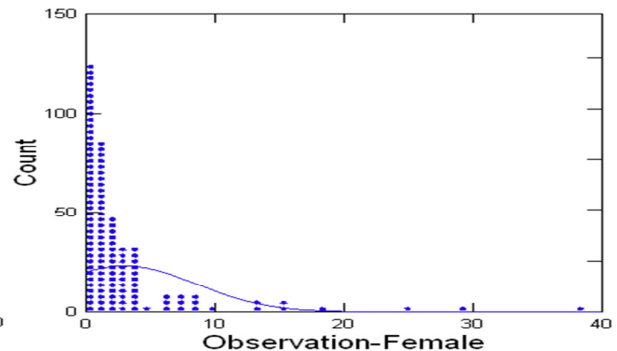


Fig.(5) Histogram Chart for T^2 -Female

Table(8) contains T^2 value of(112) observation male and (128) female. Fig(2 and 3) shows that the T^2 Control Chart of female and male. The T^2 upper control chart of male UCL is equal (13.47)from these observation there are (7) points are out of control , and for the female T^2 – upper control Chart is (13.55) it shows that (6) points are out of control. The difference between upper control of (Female and Male) is (0.08) this value is very small.

3-1- Signal-to-Noise Ratio S/N.

Determining S/N ratio and response level for female with sample size (128) and male with sample size (112) the S/N ratio for three variables (T3, T4, T.S.H) as shown in Table(3).

Table(3) Female Male Response Levels of S/N

Level	Female S/N				Male S/N			
	T3	T4	T.S.H	S/N Ratio	T3	T4	T.S.H	S/N Ratio
Level 1	-36.8	-48.59	-38.78	-19.353	-48.96	-45.03	-33.61	-18.404
Level 2	-68.01	-56.23	-66.04	-54.257	-60.25	-64.17	-75.59	-79.516
Delta	31.21	7.64	27.26	-77.822	11.29	19.14	41.98	-71.666
Rank	1	3	2	-58.200	3	2	1	-48.825
Gain	31.21	7.64	27.26		11.29	19.14	41.98	

The S/N ratio of (female and male) response level 1 is greater than level 2, the gains for all variables is positive therefor they contribute greatly to discrimination the gain of variable (T3) is (31.21) with more effect among the three variables, as in table (3), but in male case the variable (T.S.H) more effect among the three variables and the gain value of variable (T.S.H) is (41.98).

In table (3) shown that all the three variables significant and useful for (male and female).The response level (S/N) of (meal and female) is shown in Fig(6)

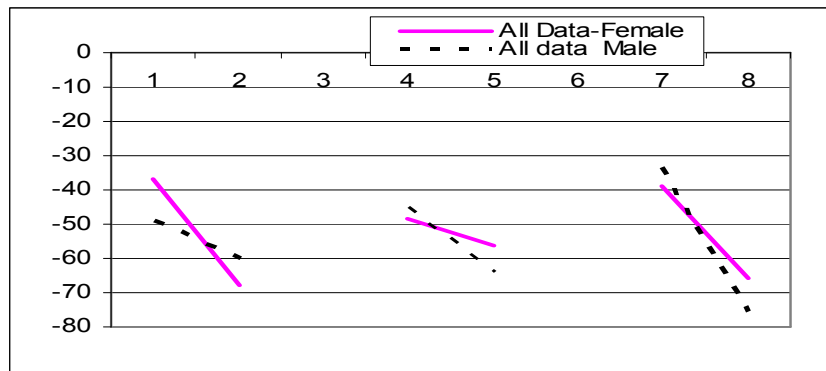


Fig.(6) Response Chart for Male and Female

3-2- Determining the S/N Ratio of (Normal and Abnormal) Data:

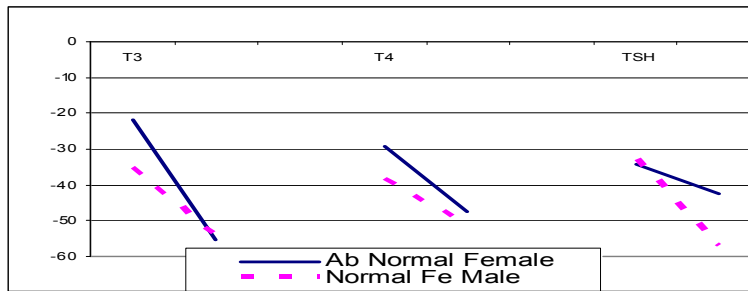
i. Female Data:

In this case determining the value of S/N ratio only for female data, with sample size (128), these samples contains normal and abnormal data. Therefor we divide data into two kinds, first type is normal with sample size(96), second type is unhealthy or (abnormal) data with sample size (32) .Abnormal data or unhealthy data are denoted that those data are not in healthy limit , Table(5) shows the response level of S/N ratio for (normal and abnormal) for female data . It is seen that the S/N ratio in Level 1 is greater than level 2. The variables contribute greatly to discrimination; they are significant and useful variables.

In the normal case the variable (T.S.H) was more effective than other variables, and the larger rank levels is (24.33) is for (T.S.H). But the S/N ratio of abnormal data the variable (T3) is more effective than other variables, and the larger rank of (T3) is (33.3). Fig (7) shows the response level of S/N ratio of (normal and abnormal) for female data.

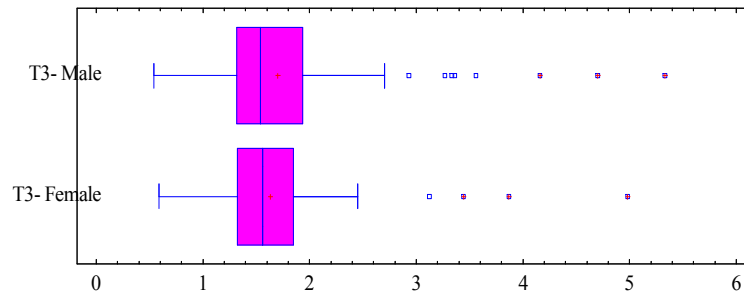
Table(5) Female – Normal and Abnormal

Female-Normal				Female-AbNormal				
Level	T3	T4	T.S.H	S/N Ratio	T3	T4	T.S.H	S/N Ratio
Level 1	-35.3	-38.3	-32.6	-16.6	-21.8	-29.3	-34.4	8.54
Level 2	-54.2	-51.2	-56.9	-53.9	-55.1	-47.6	-42.5	35
Delta	18.98	12.88	24.33	-60	33.37	18.31	8.15	50.1
Rank	2	3	1	-48.5	1	2	3	50.1
Gain	18.99	12.88	24.33		33.37	18.31	8.15	



Fig(7) Abnormal and Normal –Female

Box-and-Whisker Plot



Fig(8) Boxplot of T3 Variable for (Male and Female)

ii. Male Data:

In this case calculating the S/N ratio of male (normal and abnormal) , the sample size set with (91) abnormal and (21) normal. Table(6) shows that the level (1) of S/N ratio in normal is greater than level(2), variables are greatly useful and significant to discernment , the larger gain is (37.3) for (T.S.H) variable, and in abnormal case the first level of S/N ratio has a higher than level2 for (T4, and T.S.H) and the gain of variable (T3) is negative , the larger gain is (24.3) is for (T.S.H) variable . Clearly from the both analyses of (Female and Male) with normal data the variable (T.S.H) is more effective and variable (T3) is not a useful variable. Fig (9) shows the response level of S/N ratio of (normal and abnormal) for male data.

Table(6) Response table S/N of Male (Normal and Ab Normal)

Male-Normal				Male-AbNormal				
Level	T3	T4	T.S.H	S/N Ratio	T3	T4	T.S.H	S/N Ratio
Level 1	-49.1	-44.4	-44.2	-12.1	-32.1	-21.1	-16.7	-12.08
Level 2	-76.5	-81.2	-81.5	-86.2	-25.7	-36.7	-41	-52.01
Delta	27.4	36.8	37.3	-76.8	6.36	15.6	24.3	-30.05
Rank	3	2	1	-76.3	3	2	1	-21.32
Gain	27.4	36.8	37.3		-6.37	15.6	24.3	

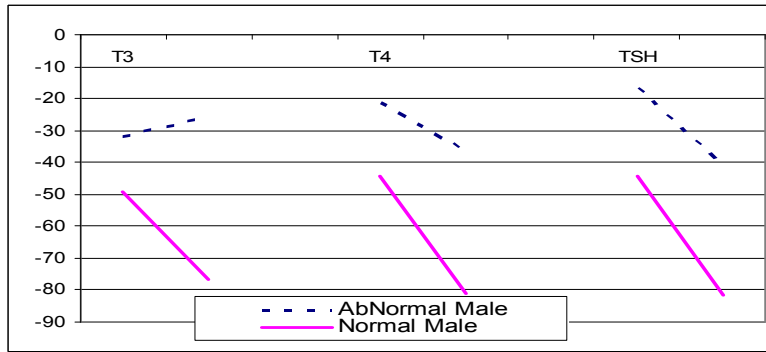


Fig.(9) SN ratio For Male (Normal and Abnormal)

Fig(10 and 11) shows the response level of normal and abnormal for (female and male), and the abnormal for (female and male). From table(5,6) for abnormal data, it shows that variable (T3) of female, and (T.S.H) of male is more effective, as shows in Fig.(7 and 9),but in Normal data the variable (T.S.H) in female and male is more effective as shows in table (5 and 6).

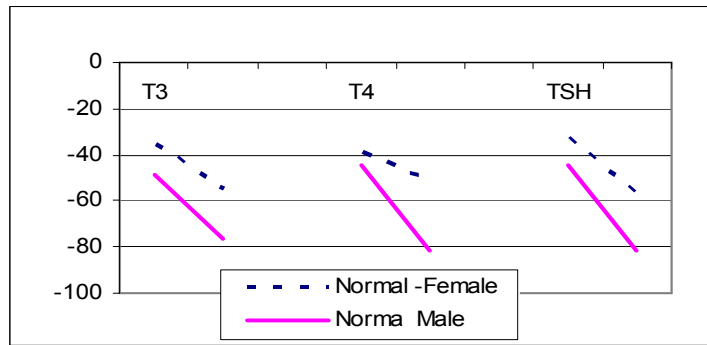


Fig.(10) SN Ratio for Normal (Female and Male)

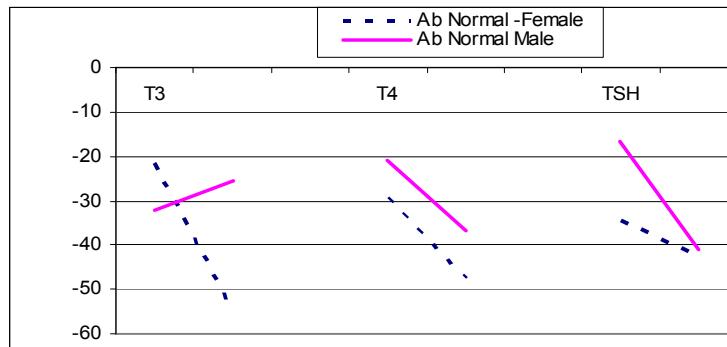


Fig.(11) -SN Ratio for Abnormal(Female and Male)

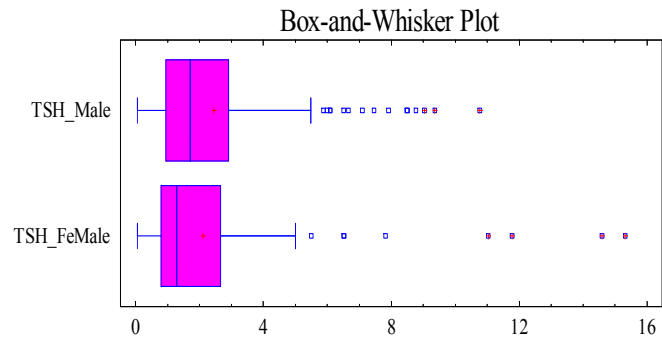


Fig.(12) –Box-Plot of TSH for (Female and Male)

CONCLUSIONS:

During this study, testing the Hotelling’s T^2 chart is the most familiar multivariate process monitoring and control procedure for monitoring the mean vector of the process and an optimization technique to estimate the value of the S/N-ratio for variables (T3,T4 and T.S.H) on the Thyroid Gland Diagnosis for (Normal and Abnormal) data (Male and Female), these three variables that indicate its importance for the diagnosis of patients .It seen that all the three variables are important and useful in the analyses to Thyroid gland diagnosis. But we conclude from the data analysis that the third variable (T.S.H) is more effective and useful with significant variable for the male in both type ,and for the female data, the first variable (T3) has more impact than variables (T4,T.S.H).

Table(7) all data collected

	sex	T3	T4	T.S.H
	X1	X2	X3	X4
1	1	1.5	93.8	1.48
2	1	2.02	88.42	5.47
3	1	1.64	73.86	1.41
4	1	1.86	112.85	2.1
5	1	1.1	76.27	0.22
6	1	2.93	99.6	2.48
7	1	3.27	136.02	0.05
8	1	1.53	143.98	0.05
9	10	1.91	89.96	1.81
10	1	1.82	95.8	0.59
11	10	1.92	91.39	1.31
12	1	3.33	193.06	0.05
13	10	1.55	78.07	1.02
14	1	2.13	84.85	0.75
15	1	1.89	109.09	1.23
16	10	1.87	86.35	0.74
17	1	1.76	87.69	1.64
18	1	1.61	91.85	3.35
19	10	2.18	97.98	3.34
20	10	1.45	86.35	1.25
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220	1	1.39	91.83	2.84
221	1	1.78	85.96	3.08
222	10	1.63	80.94	1.35
223	10	1.15	92.49	0.6
224	1	2.21	91.17	1.93
225	10	1.11	75.6	2.72
226	1	1.37	90.15	1.72
227	10	1.55	69.95	1.48
228	1	4.7	15.47	8.49
229	10	1.45	81.33	1.54
230	1	1.4	89.14	0.17
231	1	2.7	14.72	1.77
232	1	2.3	91.26	6.08
233	10	1.08	104.94	1.41
234	10	0.59	76.37	3.63
235	1	0.92	58.28	2.92
236	1	2.38	93.23	2.02
237	1	0.93	94.01	0.45
238	10	1.56	67.37	1.03
239	10	2.12	47.68	0.07
240	1	1.44	81.3	2.92

Table(8) Hotelling's T² value of Male and Female

	Hotelling's T ²		Hotelling's T ² -Female		Hotelling's T ² -Male
1	0.3166	1	0.3552	1	0.2546
2	2.0761	2	2.0896	2	0.3517
3	0.4601	3	0.6848	3	0.2831
4	1.3205	4	1.3965	4	0.4488
5	1.6726	5	2.0262	5	1.2944
6	3.8584	6	3.0822	6	0.2410
7	9.1078	7	8.6199	7	0.6997
8	7.0341	8	7.4384	8	26.038
9	0.1779	9	0.7298	9	0.2604
10	0.6125	10	24.935	10	1.5747
11	0.3122	11	1.0177	11	0.4462
12	23.2812	12	1.1272	12	0.4124
13	0.4557	13	0.1319	13	1.6419
14	0.9677	14	0.2950	14	0.1841
15	1.0738	15	7.5942	15	0.3974
16	0.5094	16	18.303	16	0.7550
17	0.0893	17	1.1235	17	0.7995
18	0.3127	18	1.5005	18	2.8085
19	1.0061	19	0.2499	19	0.2516
20	0.3104	20	0.3211	20	1.7481
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220	0.3838	108	2.9947	92	0.4680
221	0.1378	109	15.0428	93	0.5853
222	0.2224	110	2.0753	94	3.0288
223	1.3104	111	0.3199	95	1.9119
224	0.7157	112	2.4302	96	0.8538
225	0.8212	113	0.1489	97	0.9446
226	0.3453	114	0.9733	98	1.6785
227	0.6491	115	1.3451	99	0.2967
228	43.854	116	2.0631	100	0.4869
229	0.2461	117	0.3473	101	2.5562
230	0.9727	118	0.0921	102	26.232
231	15.008	119	0.5545	103	0.9106
232	3.5347	120	0.3625	104	0.1237
233	1.9781	121	38.2887	105	1.3782
234	3.0266	122	1.2091	106	0.8970
235	2.2226	123	15.7303	107	0.4486
236	1.2193	124	3.5173	108	0.1659
237	2.2059	125	2.4069	109	2.2460
238	1.0263	126	0.9448	110	3.6820
239	5.1584	127	2.1430	111	0.7288
240	0.1920	128	0.1928	112	4.7768

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